

Empty Black Holes, Firewalls, and the Origin of Bekenstein-Hawking Entropy

Mehdi Saravani,^{1,2,*} Niayesh Afshordi,^{1,2,†} and Robert B. Mann^{2,1,‡}

¹*Perimeter Institute for Theoretical Physics, 31 Caroline St. N., Waterloo, ON, N2L 2Y5, Canada*

²*Department of Physics and Astronomy, University of Waterloo, Waterloo, ON, N2L 3G1, Canada*

(Dated: December 19, 2012)

We propose a novel solution for the endpoint of gravitational collapse, in which spacetime ends (and is orbifolded) at a microscopic distance from black hole event horizons. This model is motivated by the emergence of singular event horizons in the gravitational aether theory, a semi-classical solution to the cosmological constant problem(s), and thus suggests a catastrophic breakdown of general relativity close to black hole event horizons. A similar picture emerges in *fuzzball* models of black holes in string theory, as well as the recent *firewall* proposal to resolve the information paradox. We then demonstrate that positing a surface fluid with vanishing energy density (but non-vanishing pressure) at the new boundary of spacetime, which is required by Israel junction conditions, yields a thermodynamic entropy that is identical to the Bekenstein-Hawking area law for charged rotating black holes. To our knowledge, this is the first derivation of black hole entropy which only employs *local* thermodynamics. Finally, a model for the microscopic degrees of freedom of the surface fluid (which constitute the micro-states of the black hole) is suggested, which has a finite, but Lorentz-violating, quantum field theory.

I. INTRODUCTION

General relativity (GR) predicts that the endpoint of gravitational collapse of (nonrotating and neutral) matter is a (Schwarzschild) Kerr-Newman black hole. While there might be a metric singularity at the horizon of the black hole, there is no real curvature singularity. However, there is a real curvature singularity at the centre of a black hole. Since the curvature-invariants remain small at black hole horizons, it is widely believed that black hole solutions of GR are good approximations to the “real” geometry of spacetime all the way to the singularity, except perhaps for a neighbourhood of the singularity, or at time-scales comparable to the Hawking evaporation time, where/when quantum mechanical effects become important.

Nevertheless, there are a number of arguments against the validity of the semi-classical nature of GR black hole solutions inside event horizons. If we consider GR as an effective field theory, its expected cut-off will be around the Planck energy, M_P . Quantum loop corrections should therefore be suppressed by powers of $\mathcal{O}(\mathcal{R}/M_P^2)$, which are negligible around horizons of macroscopic black holes. However, *non-perturbative* quantum effects can be big: while the tunnelling rate is suppressed by $\exp(-S_E) \ll 1$, where S_E is the Euclidean action of the instanton connecting GR solutions to other states in the full theory of quantum gravity, the number of such states is estimated to be $\sim \exp(S_{BH}) \gg 1$, with S_{BH} being the Bekenstein-Hawking entropy of black holes. Interestingly, $S_E = S_{BH}$ for Euclideanized GR black holes, and thus the non-perturbative decay of the semi-classical so-

lutions can be quite fast (i.e. much faster than the Hawking evaporation time) [1, 2]. While the nature of the end-state of gravitational collapse depends on the full phase space of the theory of quantum gravity, in the context of string theory, it has been argued that *fuzzball* solutions provide the correct multiplicity and asymptotics to represent a microscopic description of GR black hole macro-states (e.g. see [3] and references therein). While fuzzball solutions approximate GR black holes at large distances, they diverge from the classical solution (or each other) at/around the classical horizon. In particular, fuzzball solutions *do not* have any event horizons or singularities, although they contain ergo-regions, which could produce analogue of Hawking radiation. Moreover, the spacetime “ends” at a minimal spatial area comparable to that of the classical event horizon. The latter is the most significant macroscopic difference between the fuzzballs, and their semi-classical counter-parts, which we will capture below in our construction.

Very recently, a similar picture has emerged from a reconsideration of the black hole information paradox [4]: states that fall into black hole horizon are entangled with those of Hawking radiation, but unitarity implies that the end state of black hole evaporation (= early+late Hawking radiation) should be a pure state. Authors of [5] argue that (for reasons very close to Mathur’s arguments [6], or earlier arguments, e.g., in [7] and references therein) the most “conservative” resolution is to replace the horizon by a *firewall* that “burns” infalling observers. Nevertheless, many counter-arguments (and some retractions) soon followed this proposal (e.g. [8–11]).

There is also another argument, rooted in the quantum nature of gravity, for the breakdown of semi-classical spacetime at black hole horizons, which (as shown here) is further validated by the first derivation (to our knowledge) of Bekenstein-Hawking entropy based on *local* thermodynamics. The argument follows from the behaviour of an *incompressible fluid* in GR, which has been argued

*Electronic address: msaravani@pitp.ca

†Electronic address: nafshordi@pitp.ca

‡Electronic address: rbmann@uwaterloo.ca

to develop singularities close to event horizons, while simultaneously explaining the observed scale of cosmological dark energy without any fine-tuning [12]. The structure of these black hole solutions is depicted in Fig. (1).

But why an incompressible fluid? The reason comes from an attempt to solve the (old) cosmological constant problem, which is arguably the most puzzling aspect of coupling gravity to relativistic quantum mechanics [13]. Given that the natural expectation value for the vacuum of the standard model of particle physics is ~ 60 orders of magnitude heavier than the gravitational measurements of vacuum density, it is reasonable to entertain an alternative theory of gravity where the standard model vacuum decouples from gravity. Such a theory could be realized by coupling gravity to the traceless part of the quantum mechanical energy-momentum tensor. However, the consistency/covariance of gravitational field equations then requires introducing an auxiliary fluid, the so-called *gravitational aether* [14]. The simplest model for gravitational aether is an incompressible fluid (with vanishing energy density, but non-vanishing pressure), which is currently consistent with all cosmological, astrophysical, and precision tests of gravity [15, 16]:

$$\begin{aligned} \frac{3}{32\pi G_N} G_{\mu\nu} &= T_{\mu\nu} - \frac{1}{4} T^\alpha_\alpha g_{\mu\nu} + T'_{\mu\nu}, \\ T'_{\mu\nu} &= p'(u'_\mu u'_\nu + g_{\mu\nu}), \quad T'^{\mu\nu}_{;\nu} = 0, \end{aligned} \quad (1)$$

where G_N is Newton's constant, $T_{\mu\nu}$ is the matter energy momentum tensor and $T'_{\mu\nu}$ is the incompressible gravitational aether fluid. In vacuum, the theory reduces to GR coupled to an incompressible fluid.

Motivated by the existence of singularities or minimum area surfaces close to black hole event horizons in the models mentioned above, we propose a new model for black holes in which spacetime ends, and is orbifolded, at a microscopic distance from black hole horizons. In this model, a black hole is a “bubble of nothing” (reminiscent of [17]). We then show that putting a (2+1 dimensional) surface fluid with vanishing density (i.e. incompressible) at the new boundary, which is required by Israel junction conditions, gives a thermodynamic entropy identical to the Bekenstein-Hawking entropy. This work also suggests an analogy between black holes in 3+1 dimensions and 2+1 dimensional incompressible fluids, which have been studied earlier in the context of holography [18, 19].

The outline of our paper is as follows. Section II is devoted to our new model of empty black holes and the resulting local derivation of the Bekenstein-Hawking entropy for charged rotating empty black holes. We then present a toy microscopic description for the surface fluid with (near-)vanishing density in Section III. Finally, Section IV concludes the paper.

II. EMPTY BLACK HOLES AND THE BEKENSTEIN-HAWKING ENTROPY

In the previous section, we discussed motivations to posit a minimum area surface close to/at black hole horizons. As a result, we may model a black hole as a hole in spacetime – a bubble of nothing – and end spacetime at a microscopic distance from the putative horizon (at stretched horizon). It will be the responsibility of a full quantum gravity theory to resolve this singularity.

First, we will explain the empty black hole model for spherically symmetric black holes (Schwarzschild) and derive Bekenstein-Hawking entropy. Then, we will extend the model to the most general black holes in four dimensions, i.e. Kerr-Newman black holes.

A. Schwarzschild Black Holes

Once there is a boundary in spacetime, we need to specify a boundary condition. We impose radial Z_2 symmetry at the boundary, as it is a natural boundary condition for a spherically symmetric solution. This Z_2 boundary condition also appears in the membrane paradigm for black holes [20].

Consider a static spherically symmetric spacetime which, in general, has the following line element

$$ds^2 = -N^2(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2, \quad (2)$$

where $d\Omega^2 \equiv d\theta^2 + \sin^2(\theta)d\phi^2$ is the line element of 2-sphere. Here $N(r)$ and $f(r)$ are arbitrary functions satisfying GR field equations in the bulk (for Schwarzschild $N^2(r) = f(r) = 1 - \frac{2m}{r}$.) If there were no boundary, then we would have a horizon at $r = r_0$ where $N(r_0) = 0$. However, in this model, spacetime ends at a microscopic distance $r = r^* > r_0$ from the horizon.

We assume there is a thin layer of fluid sitting at the boundary ($r = r^*$), with the following energy-momentum tensor

$$\mathcal{T}_{ab} = (\Sigma + \Pi)U_a U_b + \Pi h_{ab}, \quad (3)$$

where \mathcal{T}_{ab} is the surface energy-momentum tensor, Σ is surface energy density, Π is surface pressure, U_a is the fluid 3-velocity, h_{ab} is the induced metric on the hypersurface $r = r^*$ and $a, b \in \{t, \theta, \phi\}$. In fact, as we will show later, imposing radial Z_2 symmetry on the boundary *requires* the existence of this fluid.

For a general hypersurface S_r defined as $r = \text{constant} > r^*$, the line element on S_r can be written as

$$dl_3^2 = -N^2(r)dt^2 + r^2 d\Omega^2. \quad (4)$$

So we obtain

$$h_{ab} = \text{diag}(-N^2, r^2, r^2 \sin^2 \theta), \quad (5)$$

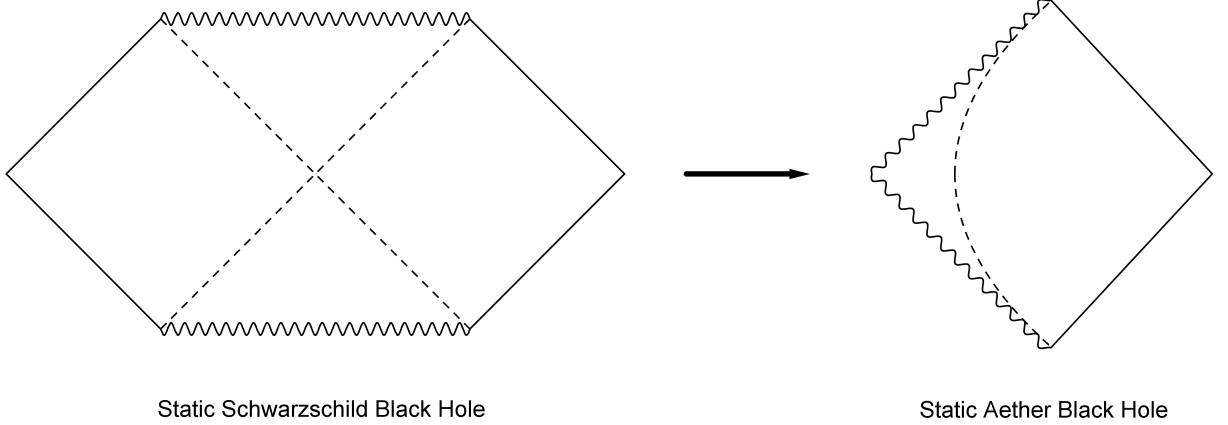


FIG. 1: Comparison of the causal diagrams for the static Schwarzschild black hole, and the static black holes in gravitational aether [12]. In both diagrams, the solid lines depict null infinities, while the squiggly lines are singularities, and g^{rr} vanishes on dotted lines. However, the latter is a null surface in Schwarzschild BH which coincides with event horizon, while it is time-like in the aether BH and corresponds to a throat or minimal area surface. Moreover, while the singularity is space-like and sits at zero area deep inside the horizon in the Schwarzschild BH, it is null in the Aether BH and sits at finite area, roughly a Planck length inside the throat. The latter assumption is the key ingredient for aether pressure to match the observed dark energy pressure for astrophysical BH masses [12].

and

$$K_{ab} = \text{diag}(-N'Nf^{1/2}, rf^{1/2}, rf^{1/2}\sin^2\theta), \quad (6)$$

for the extrinsic curvature $K_{ab} = \frac{1}{2}h_a^c h_b^d \nabla_c n_d$ of the hypersurface, where $N' \equiv \frac{dN}{dr}$.

We now employ the Israel junction conditions

$$[h_{ab}] = 0 \quad (7)$$

$$[K_{ab}] - [K]h_{ab} = -8\pi\mathcal{T}_{ab}, \quad (8)$$

where $[A] \equiv A(r^+) - A(r^-)$ is the discontinuity of $A(r)$ across the hypersurface S_r . Using equations (3), (6), (7) and (8) for a static solution (U_a having only a non-zero temporal component), we obtain

$$[N] = 0, \quad (9)$$

$$4\pi\Sigma = -\left[\frac{f^{1/2}}{r}\right], \quad (10)$$

$$8\pi\Pi = \left[\frac{N'}{N}f^{1/2}\right] + \left[\frac{f^{1/2}}{r}\right] \quad (11)$$

for a hypersurface of radius r . In particular, we could write the previous junction conditions at $r = r^*$. However, S_{r^*} is the boundary of spacetime, and the discontinuity of functions across S_{r^*} is not defined. Despite this fact, we can show (see Appendix A for details) that imposing radial Z_2 symmetry (for time-like boundaries) modifies (7) and (8) to

$$K_{ab} - Kh_{ab} = -8\pi\mathcal{T}_{ab}. \quad (12)$$

As a result, we get

$$4\pi\Sigma = -\frac{\sqrt{f(r^*)}}{r^*}, \quad (13)$$

$$8\pi\Pi = \frac{N'(r^*)}{N(r^*)}\sqrt{f(r^*)} + \frac{\sqrt{f(r^*)}}{r^*}. \quad (14)$$

Note that for Schwarzschild metric, in the limit $r^* \rightarrow r_0$ equation (13) gives $\Sigma = 0$.

Since the surface fluid is at constant radius it consequently sees the thermal radiation due to its acceleration (Unruh effect [21]; we will further justify this choice in IV). Assuming the fluid is in thermal equilibrium with the Unruh radiation, its temperature is fixed by the temperature of the radiation in the fluid's vicinity, and so

$$T(r^*) = T_{\text{Unruh}} = \frac{1}{2\pi} \frac{N'(r^*)}{N(r^*)} \sqrt{f(r^*)}. \quad (15)$$

Note that the fluid pressure (14) and temperature (15) diverge in the limit $r^* \rightarrow r_0$.

We now have everything to calculate the entropy of the surface fluid. Assuming local thermodynamic equilibrium (LTE) at zero chemical potential (which is expected at high temperatures), the entropy per unit area of this fluid is given by

$$s = \frac{\Sigma + \Pi}{T} \quad (16)$$

yielding

$$s = \frac{1}{4}, \quad (17)$$

which is the same as Bekenstein-Hawking entropy.

B. Kerr-Newman Black Holes

We can also extend this model to charged rotating black holes. As shown in Appendix B (see Eq. B1), the near horizon geometry of a Kerr-Newman black hole is

$$ds^2 = -\Gamma_+ \lambda^2 d\tau^2 + \Gamma_+ d\lambda^2 + \Gamma_+ d\theta^2 + \frac{\sin^2 \theta}{\Gamma_+} d\psi^2,$$

where $\Gamma_+ \equiv r_+^2 + a^2 \cos^2 \theta$ and $\lambda = 0$ corresponds to the horizon of black hole. Similar to the Schwarzschild case, we end the spacetime at the stretched horizon ($\lambda = \lambda^* > 0$, taking the limit $\lambda^* \rightarrow 0$) and impose Z_2 boundary condition at the new boundary.

The Z_2 symmetric boundary requires that the extrinsic curvature of the boundary should satisfy (12). Expressing the induced metric on the boundary in (τ, θ, ψ) coordinates we obtain

$$h_{ab} = \text{diag}(-\Gamma_+ \lambda^{*2}, \Gamma_+, \frac{\sin^2 \theta}{\Gamma_+}) \quad (18)$$

where the normal vector to the hypersurface $\lambda = \lambda^*$ is

$$n_\mu = \sqrt{\Gamma_+} (0, 1, 0, 0), \quad (19)$$

$$n^\mu = \frac{1}{\sqrt{\Gamma_+}} (0, 1, 0, 0). \quad (20)$$

Consequently we find

$$K_{ab} = \frac{1}{2\sqrt{\Gamma_+}} (\frac{\partial h_{ab}}{\partial \lambda})_{\lambda^*} = \text{diag}(-\sqrt{\Gamma_+} \lambda^*, 0, 0), \quad (21)$$

$$K = K_{ab} h^{ab} = \frac{1}{\sqrt{\Gamma_+ \lambda^*}}. \quad (22)$$

Using (3) with

$$U_a = \sqrt{\Gamma_+ \lambda^*} (1, 0, 0) \quad (23)$$

(so that $U^a U^b h_{ab} = -1$), we find

$$\mathcal{T}_{ab} = \text{diag}(\Sigma \Gamma_+ \lambda^{*2}, \Pi \Gamma_+, \Pi \frac{\sin^2 \theta}{\Gamma_+}). \quad (24)$$

Note that this means that the fluid is comoving with the black hole, where $\sqrt{\Gamma_+ \lambda^*}$ is the normalization factor. Recall that we are working in (τ, θ, ψ) coordinates, so a zero velocity component in the direction of ψ means that the fluid is rotating with angular frequency Ω in the direction of ϕ .

Using (12), we get

$$\Sigma = 0, \quad \Pi = \frac{1}{8\pi \sqrt{\Gamma_+ \lambda^*}}. \quad (25)$$

As before, we assume equilibrium, and so the temperature of the surface fluid is fixed by the temperature of Unruh radiation. Since the acceleration of the fluid is

$$a = \frac{1}{\sqrt{\Gamma_+ \lambda^*}}, \quad (26)$$

we find

$$T = a/2\pi = \frac{1}{2\pi \sqrt{\Gamma_+ \lambda^*}}, \quad (27)$$

for the Unruh temperature. Note that this angle-dependent temperature is the same as blue-shifted Hawking temperature of Kerr-Newman black hole for a co-rotating observer at the position of the boundary. Finally, the entropy per unit area of the fluid will be

$$s = \frac{\Sigma + \Pi}{T} = \frac{1}{4}. \quad (28)$$

III. MICROSCOPIC DERIVATION OF AN INCOMPRESSIBLE FLUID

In the previous section, we showed that a stationary solution requires the existence of an incompressible surface fluid on the boundary of an empty black hole. In this section we show that a class of dispersion relations for matter can give rise to a nearly incompressible fluid at high energies (by nearly incompressible, we mean that the pressure of the fluid is much greater than its energy density).

For a thermal gas of bosons/fermions at temperature T , energy density ρ and pressure \mathcal{P} are as follows

$$\rho(T) = g \int \frac{d^3 p}{(2\pi)^3} E(p) n(p) = \langle E \rangle, \quad (29)$$

$$\mathcal{P}(T) = \frac{1}{3} g \int \frac{d^3 p}{(2\pi)^3} p \frac{dE}{dp} n(p) = \frac{1}{3} \langle p \frac{dE}{dp} \rangle, \quad (30)$$

where g is the degeneracy factor, $E(p)$ is dispersion relation (relation between energy and momentum of a particle) and

$$n(p) = \frac{1}{e^{E(p)/T} \pm 1}, \quad (31)$$

where for simplicity we set the chemical potential $\mu = 0$ (+ for fermions, - for bosons.)

Once we specify a dispersion relation, we are able to compute the energy density and pressure of a fluid of these particles. While particles with mass m at low energies satisfy the Lorentzian dispersion relation $E^2 = p^2 + m^2$, as we argue below, there are reasons to believe the energy-momentum relation might be modified at high energies (e.g. [22]).

For example, consider the dispersion relation [23]

$$E^2 = \frac{p^2}{1 - p^2/\Lambda^2}, \quad (32)$$

which reduces to the Lorentzian dispersion relation (with $m = 0$) at low energies ($p \ll \Lambda$), while it deviates from Lorentzian dispersion relation at high energies. Indeed, energy becomes infinite for a finite value of momentum. In Fig.2 we depict the equation of state variable w (pressure over density) of a fluid obeying the dispersion relation (32), as a function of temperature. We see that

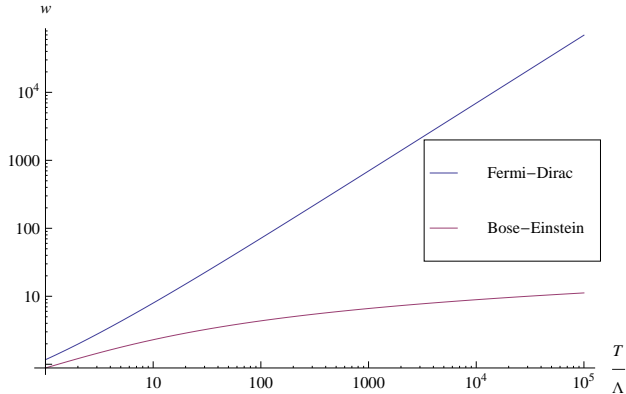


FIG. 2: Equation of State variable ($w = \frac{P}{\rho}$) as a function of Temperature.

w grows as temperature increases, and approaches infinity in the limit of infinite compression, expected for the surface fluid.

Generically, fluid of particles with a dispersion relation in which energy as a function of momentum grows faster than any power law becomes a near incompressible fluid at high temperatures. We will show this by an example. Consider the following dispersion relation

$$E^2 = p^2 + \frac{p^{2l}}{\Lambda^{2(l-1)}}. \quad (33)$$

At low temperatures ($T \ll \Lambda$), energies of particles are effectively equal to their momentum $E \approx p$, and according to (29) and (30)

$$w = \frac{P}{\rho} \approx \frac{1}{3}. \quad (34)$$

However, at high temperatures ($T \gg \Lambda$) the dispersion relation changes to $E \approx \frac{p^l}{\Lambda^{l-1}}$ and, as a result

$$w \approx \frac{l}{3}. \quad (35)$$

However, if dispersion relation grows faster than any power law for large p , then l increases as temperature increases and goes to infinity as temperature goes to infinity. Consequently, according to (35), w increases unboundedly with temperature.

On the other hand, the dispersion relation (32) regulates the UV infinities of quantum field theory, since there is a maximum momentum for any particle. We show here the argument for a real scalar field ϕ . In the interaction

picture, ϕ can be expanded in terms of creation and annihilation operators as

$$\phi(x) = \int^\Lambda \frac{d^3\vec{p}}{(2\pi)^3} \frac{1}{\sqrt{2E_{\vec{p}}}} (a_{\vec{p}} e^{ip \cdot x} + a_{\vec{p}}^\dagger e^{-ip \cdot x}), \quad (36)$$

where $p \cdot x \equiv -p^0 t + \vec{p} \cdot \vec{x}$, $p^0 = E_{\vec{p}} (\equiv E(p))$ and $[a_{\vec{p}}, a_{\vec{q}}^\dagger] = (2\pi)^3 \delta^{(3)}(\vec{p} - \vec{q})$. Using the above expansion, we obtain

$$\begin{aligned} G_F(x-y) &= \langle 0 | T \phi(x) \phi(y) | 0 \rangle \\ &= i \int \frac{d^4 p}{(2\pi)^4} \frac{\theta(\Lambda - |\vec{p}|)}{(p^0)^2 - (E_{\vec{p}})^2 + i\epsilon} e^{ip \cdot (x-y)}. \end{aligned} \quad (37)$$

For canonical interactions, loop infinities originate from integration over products of Feynman Green's functions. However, since there is a cut off for spatial part of momentum, loop corrections of quantum field theory will be finite.

IV. DISCUSSION AND CONCLUSIONS

We have shown that a surface fluid at Unruh temperature with vanishing energy density on the stretched horizon of a black hole has the same thermodynamic entropy as Bekenstein-Hawking entropy. The surface fluid is the result of ending (or orbifolding) spacetime at the stretched horizon and replacing it with a Z_2 symmetric boundary. We therefore conjecture that the microstates of a black hole are those of the surface fluid at the stretched horizon. We emphasize that this description is very similar to the traditional membrane paradigm for black hole horizons [20]. However, in our description, the membrane properties are physical and result from condensation of accreted matter onto a *physical* membrane, while they arise only as a mathematical analogy in the traditional membrane paradigm (e.g. the pressure in [24]). Fig. (3) compares the expected causal diagrams for collapse of standard and aether BH's.

But why Unruh temperature? Our derivation of black hole entropy crucially relied on assuming that the surface fluid is heated up to the Unruh temperature [21], set by the acceleration of the observers on the stretched horizon. However, the derivation of Unruh temperature relies on a (locally) Minkowski vacuum at the vicinity of the horizon, which is exactly what we are trying to replace in our picture. For example, one can construct the ground state for the space-time outside the stretched horizon (the Boulware vacuum) [25], which has zero thermodynamic temperature. However, here we should differentiate between *absolute* and *local* thermal equilibrium. The former has an exact definition that fixes the density matrix of the system in terms of the Hamiltonian, and a global temperature T : $\rho \propto \exp(-H/T)$. However, the latter is only an approximation, valid in the fluid limit, and allows spatial/temporal variation of temperature, $T(\mathbf{x}, t)$. Indeed, this is the temperature that, e.g., a thermometer would

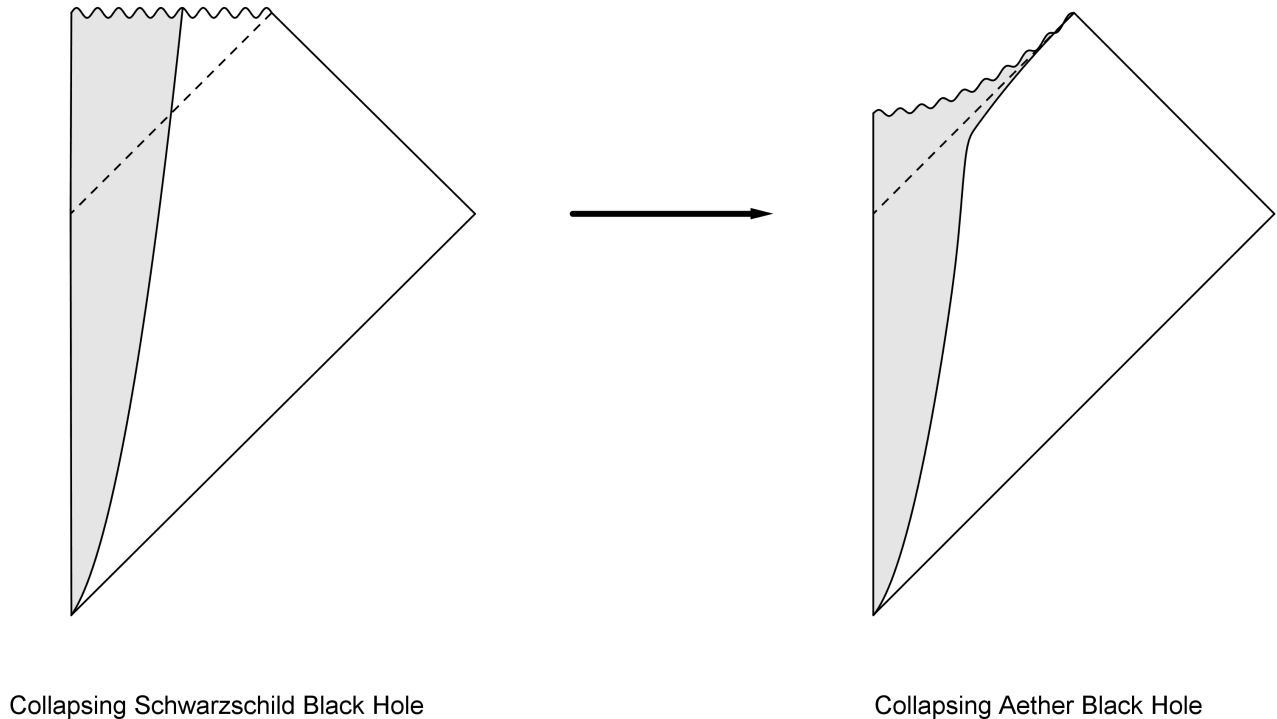


FIG. 3: Comparison of the causal diagrams for a collapsing Schwarzschild black hole, and our proposed picture for collapsing black holes in gravitational aether or firewall scenarios. In both diagrams, the dotted lines depict classical event horizons, while the squiggly lines are singularities. The black area correspond to the collapsing star. While in the Schwarzschild BH, the singularity is space-like and deep inside the horizon, in the aether/firewall case it approaches the horizon and becomes null asymptotically. The accreted material smoothly crosses the Schwarzschild horizon, but it condenses into Planckian densities just inside the horizon of the aether BH.

measure. Here, we argue that this *local* equilibrium temperature set by the Unruh value $= a/(2\pi)$, independent of the choice of vacuum in the black hole space-time.

It turns out that there is a completely classical description of the Unruh (or blue-shifted Hawking) thermal spectrum, simply based on propagation of wave-packets in near-horizon geometries (to our knowledge, this was first outlined in [26]): The exponential redshift of any wave-packet that originates near the horizon of a Kerr-Newman black hole, converts it into a near-thermal spectrum at infinity at exactly the Hawking temperature. Although the classical propagation doesn't fix the amplitude of the Hawking radiation, the requirement of equilibrium with a thermal bath sets the amplitude to that of a blackbody spectrum. Therefore, we see that the assumption of an atmosphere in *local thermal equilibrium in the near-horizon geometry* sets the local temperature to the Unruh (or blue-shifted Hawking) value, *independent of the quantum state, or its regularity at the horizon*.

We further argue that our derivation of Bekenstein-

Hawking entropy is the first truly *local* description of black hole micro-states. For example, models for the area law based on entanglement entropy, or string theory (e.g., in fuzzball proposal) are inherently non-local: the total entanglement entropy depends on an uncertain cut-off, and only reproduces the area law up to an order unity factor [27]. Computing the change in the entanglement entropy can reproduce the correct change in the area law (assuming Einstein equations) [28], but does NOT localize the total entropy on the surface. Similarly, the counting of string theory fuzzball solutions involves summing over states with no classical 4d counterpart (e.g., [1]).

Another proposal for the endpoint of gravitational collapse, which is similar to our model, is the gravastar [29]. Exterior of a gravastar is Schwarzschild geometry, whereas the interior is replaced by de Sitter spacetime that is matched to the exterior via a thin shell of stiff fluid (a fluid with $p = \rho$.) However, the entropy of a gravastar is much less than the Bekenstein-Hawking entropy.

Additionally, Bekenstein-Hawking entropy can be derived as the thermodynamic entropy of a stiff fluid in the presence of a black hole [30, 31]. In this model, the horizon is replaced by a high density thin shell of stiff fluid. However, this solution suffers from the presence of a point-like naked singularity with negative mass at the center. In this context, an interesting question for future study would be to consider the gravitational collapse of matter (e.g. a ball of dust) in the context of gravitational aether to see how the end state is (classically) approached.

While the list of motivations for truncating the classical spacetime at black horizons has grown manyfold over the past few years, here we note the recurrence of the notion of *incompressibility*, in our exposition. Indeed, incompressibility is no longer considered a mathematical novelty (e.g. [18, 19, 32–34]), and has appeared, and re-appeared in different physical contexts. In particular, appearance of an incompressible fluid in both bulk *and* boundary of gravitational aether black holes is suggestive of the continuity of the underlying microscopic phenomenon. The obvious difference between the two fluids, however, is that the boundary fluid carries a finite entropy, while the gravitational aether has a degenerate phase space (and thus zero or negligible entropy). Nevertheless, it is not clear whether this difference might be an artifact of the surface condensation, or rather point to different origins of the two fluids. A related puzzle is why Bekenstein-Hawking entropy is localized on the surface (at least at the classical level), while the bulk

incompressible fluid can cause non-local interactions.

Furthermore, one might ask what type of horizons are expected to develop singularities. Do we expect singularities for Rindler or de Sitter horizons? For example, authors of [5] argue singularities (or firewalls) only occur in “old” horizons, at a fraction of their evaporation time, which never happens for de Sitter or Rindler horizons.

Even more speculative, but most exciting, is the possibility of directly probing quantum gravitational effects by precision studies of astrophysical horizons (e.g. [35]). Given that the conditions of big bang is now replicated close to black hole horizons (and not deep inside them), the observational constraints on the early universe might now be applied to the microscopic structure of horizons, or conversely, black hole observations can potentially constrain the nature of cosmological big bang.

Acknowledgement: The authors would like to thank Siavash Aslanbeigi, Eugenio Bianchi, Samir Mathur, Paul McFadden, Rafael Sorkin, and Dejan Stojkovic for invaluable discussions. This work was supported by the Natural Science and Engineering Research Council of Canada, the University of Waterloo and the Perimeter Institute for Theoretical Physics. Research at the Perimeter Institute is supported by the Government of Canada through Industry Canada and by the Province of Ontario through the Ministry of Research & Innovation.

-
- [1] S. D. Mathur, Gen.Rel.Grav. **42**, 113 (2010), arXiv:0805.3716 [hep-th] .
 - [2] S. D. Mathur, Int.J.Mod.Phys. **D18**, 2215 (2009), arXiv:0905.4483 [hep-th] .
 - [3] S. D. Mathur, (2012), arXiv:1205.0776 [hep-th] .
 - [4] S. Hawking, Phys.Rev. **D14**, 2460 (1976).
 - [5] A. Almheiri, D. Marolf, J. Polchinski, and J. Sully, (2012), arXiv:1207.3123 [hep-th] .
 - [6] S. D. Mathur, Class.Quant.Grav. **26**, 224001 (2009), arXiv:0909.1038 [hep-th] .
 - [7] R. D. Sorkin, Stud.Hist.Philos.Mod.Phys. **36**, 291 (2005), arXiv:hep-th/0504037 [hep-th] .
 - [8] L. Susskind, (2012), arXiv:1207.4090 [hep-th] .
 - [9] R. Bousso, (2012), arXiv:1207.5192 [hep-th] .
 - [10] D. Harlow, (2012), arXiv:1207.6243 [hep-th] .
 - [11] Y. Nomura, J. Varela, and S. J. Weinberg, (2012), arXiv:1207.6626 [hep-th] .
 - [12] C. Prescod-Weinstein, N. Afshordi, M. L. Balogh, N. Afshordi, and M. L. Balogh, Phys.Rev. **D80**, 043513 (2009), arXiv:0905.3551 [astro-ph.CO] .
 - [13] S. Weinberg, Rev.Mod.Phys. **61**, 1 (1989).
 - [14] N. Afshordi, (2008), arXiv:0807.2639 [astro-ph] .
 - [15] F. Kamiab and N. Afshordi, Phys.Rev. **D84**, 063011 (2011), arXiv:1104.5704 [astro-ph.CO] .
 - [16] S. Aslanbeigi, G. Robbers, B. Z. Foster, K. Kohri, and N. Afshordi, Phys.Rev. **D84**, 103522 (2011), arXiv:1106.3955 [astro-ph.CO] .
 - [17] E. Witten, Nuclear Physics B **195**, 481 (1982).
 - [18] G. Compere, P. McFadden, K. Skenderis, and M. Taylor, JHEP **03**, 076 (2012), arXiv:1201.2678 [hep-th] .
 - [19] I. Bredberg and A. Strominger, JHEP **1205**, 043 (2012), arXiv:1106.3084 [hep-th] .
 - [20] R. H. Price and K. S. Thorne, Scientific American **258**, 69 (1988).
 - [21] W. Unruh, Phys.Rev. **D14**, 870 (1976).
 - [22] P. Horava, Phys.Rev. **D79**, 084008 (2009), arXiv:0901.3775 [hep-th] .
 - [23] J. Magueijo and L. Smolin, Phys.Rev. **D67**, 044017 (2003), arXiv:gr-qc/0207085 [gr-qc] .
 - [24] S. Kolekar and T. Padmanabhan, Phys.Rev. **D85**, 024004 (2012), arXiv:1109.5353 [gr-qc] .
 - [25] D. G. Boulware, Phys.Rev. **D11**, 1404 (1975).
 - [26] M. Nouri-Zonoz and T. Padmanabhan, (1998), arXiv:gr-qc/9812088 [gr-qc] .
 - [27] L. Bombelli, R. K. Koul, J. Lee, and R. D. Sorkin, Phys.Rev. **D34**, 373 (1986).
 - [28] E. Bianchi, (2012), arXiv:1211.0522 [gr-qc] .
 - [29] P. O. Mazur and E. Mottola, (2001), arXiv:gr-qc/0109035 [gr-qc] .
 - [30] W. H. Zurek and D. N. Page, Phys. Rev. D **29**, 628 (1984).
 - [31] G. 't Hooft, Nucl.Phys.Proc.Suppl. **68**, 174 (1998), arXiv:gr-qc/9706058 [gr-qc] .
 - [32] N. Afshordi, D. J. Chung, and G. Geshnizjani, Phys.Rev.

- D75**, 083513 (2007), arXiv:hep-th/0609150 [hep-th] .
- [33] N. Afshordi, Phys.Rev. **D80**, 081502 (2009), arXiv:0907.5201 [hep-th] .
- [34] I. Bredberg, C. Keeler, V. Lysov, and A. Strominger, JHEP **1103**, 141 (2011), arXiv:1006.1902 [hep-th] .
- [35] A. E. Broderick and R. Narayan, Class.Quant.Grav. **24**, 659 (2007), arXiv:gr-qc/0701154 [GR-QC] .
- [36] W. Israel, Il Nuovo Cimento B 19651970 **571**, 1 (1966).

Appendix A: Israel Junction Condition at Z_2 Symmetric Boundary

In order to get junction conditions at a boundary with radial Z_2 symmetry, we will use the same technique as [36]. The metric (2) can be written in terms of proper radial distance as

$$ds^2 = -N(\lambda)^2 dt^2 + d\lambda^2 + r(\lambda)^2 d\Omega^2, \quad (\text{A1})$$

where

$$\lambda(r) = \int_{r^*}^r \frac{dr'}{\sqrt{f(r')}}.$$

Radial Z_2 symmetry thus implies that for points at $\lambda = 0$, the metric can be locally extended to negative values of λ , such that

$$N(\lambda) = N(-\lambda) \quad (\text{A2})$$

$$r(\lambda) = r(-\lambda). \quad (\text{A3})$$

Also, if there is a thin layer of fluid between $\lambda = 0$ and $\lambda = \epsilon$, radial Z_2 symmetry will require the existence of a fluid with the same energy momentum tensor between $\lambda = 0$ and $\lambda = -\epsilon$.

If K_{ab} and ${}^3\mathcal{R}_{ab}$ represent the extrinsic curvature and intrinsic curvature of a surface with constant λ (which means surface with constant r), respectively, then in (A1) coordinates [36]

$$\begin{aligned} g_{ab} &= h_{ab} = \text{diag}(-N^2, r^2, r^2 \sin^2 \theta) \\ K_{ab} &= \frac{1}{2} \frac{\partial g_{ab}}{\partial \lambda}, \\ \mathcal{R}_{ab} &= - \frac{\partial K_{ab}}{\partial \lambda} + Z_{ab}, \end{aligned} \quad (\text{A4})$$

where

$$Z_{ab} = {}^3\mathcal{R}_{ab} - K K_{ab} + 2K_a^c K_{cb}.$$

Using the Einstein field equation

$$R_{\mu\nu} = 8\pi(T_{\mu\nu} - \frac{1}{2}Tg_{\mu\nu}),$$

and integrating through the layer (from $\lambda = -\epsilon$ to $\lambda = \epsilon$), we obtain

$$K_{ab}(\epsilon) - K_{ab}(-\epsilon) - \int_{-\epsilon}^{\epsilon} d\lambda Z_{ab} = -8\pi \int_{-\epsilon}^{\epsilon} d\lambda (T_{ab} - \frac{1}{2}g_{ab}T). \quad (\text{A5})$$

Imposing Z_2 symmetry and taking the limit $\epsilon \rightarrow 0$ gives

$$\mathcal{T}_{ab} \equiv \int_0^\epsilon d\lambda T_{ab} = \frac{1}{2} \int_{-\epsilon}^\epsilon d\lambda T_{ab}.$$

Also, (A2),(A3) and (A4) give

$$\lim_{\epsilon \rightarrow 0} [K_{ab}(\epsilon) - K_{ab}(-\epsilon)] = 2K_{ab} \text{ at } r = r^*.$$

In addition, if K_{ab} remains bounded, then

$$\lim_{\epsilon \rightarrow 0} \int_{-\epsilon}^{\epsilon} d\lambda Z_{ab} = 0.$$

In (A1) coordinates $g_{ab} = h_{ab}$. Consequently, (A5) results in

$$-8\pi(\mathcal{T}_{ab} - \frac{1}{2}h_{ab}\mathcal{T}) = K_{ab}. \quad (\text{A6})$$

The above equation has been derived in a particular coordinate for boundary. However, it is a tensorial equation and accordingly, it is valid in any coordinate for boundary. Taking trace of (A6) and expressing T in terms of $K \equiv K_{ab}h^{ab}$, we get

$$K_{ab} - Kh_{ab} = -8\pi\mathcal{T}_{ab}$$

We argued that for the special case of spherical symmetry, (A2) and (A3) are the conditions required to have Z_2 symmetric boundary. This definition can be extended to more general spacetimes.

Let's start with an intuitive definition for Z_2 symmetry. A spacetime (\mathcal{M}, g) has (local) Z_2 symmetry with respect to a hypersurface S , which divides spacetime into two parts (\mathcal{M}^+, g^+) and (\mathcal{M}^-, g^-) , if local observers on S cannot distinguish between \mathcal{M}^+ and \mathcal{M}^- . This means if they move perpendicular to S (along normal vector n to S) into \mathcal{M}^+ or \mathcal{M}^- , they will see the same geometry. In mathematical language, it means

$$\mathcal{L}_n g_{\mu\nu}^+ = \mathcal{L}_{-n} g_{\mu\nu}^-, \quad (\text{A7})$$

where \mathcal{L}_n is Lie derivative with respect to n . In particular, it results

$$K_{ab}^+ = -K_{ab}^-,$$

and Israel junction condition (8) for Z_2 symmetric hypersurface (with space-like normal vector) gives

$$2(K_{ab} - Kh_{ab}) = -8\pi\mathcal{T}_{ab}. \quad (\text{A8})$$

However, if S is the boundary of spacetime, the situation is a bit different, since S does not divide spacetime into two parts. In this case, Z_2 symmetric *boundary* means that we glue a copy of spacetime \mathcal{M} to itself through S (it means that S acts like a mirror.) Now, S has divided the whole spacetime $(\mathcal{M} + \mathcal{M})$ into two parts and condition (A7) has been satisfied. However, we must multiply the right hand side of (A8) by a factor of two, because there is also a copy of the surface fluid on the other side (as we showed concretely for spherically symmetric spacetimes.)

As a result, we obtain

$$K_{ab} - Kh_{ab} = -8\pi\mathcal{T}_{ab}, \quad (\text{A9})$$

for Z_2 symmetric boundaries. Indeed, equation (A9) can be used as a definition for Z_2 symmetric boundary.

Appendix B: Near Horizon Geometry of Kerr-Newman Black Hole

The Kerr-Newman metric describes the geometry of a black hole with angular momentum J and charge Q . This metric can be written as

$$\begin{aligned} ds^2 &= g_{tt}dt^2 + g_{rr}dr^2 + g_{\theta\theta}d\theta^2 + g_{\phi\phi}d\phi^2 + 2g_{t\phi}dtd\phi \\ &= (g_{tt} - \frac{g_{t\phi}^2}{g_{\phi\phi}})dt^2 + g_{rr}dr^2 + g_{\theta\theta}d\theta^2 + g_{\phi\phi}(d\phi + \frac{g_{t\phi}}{g_{\phi\phi}}dt)^2, \end{aligned}$$

where

$$\begin{aligned} g_{tt} &= -(1 - \frac{2mr - r_Q^2}{\Gamma}), \\ g_{rr} &= \frac{\Gamma}{\Delta}, \\ g_{\theta\theta} &= \Gamma, \\ g_{\phi\phi} &= \sin^2\theta \left(r^2 + a^2 + \frac{(2mr - r_Q^2)a^2 \sin^2\theta}{\Gamma} \right), \\ g_{t\phi} &= -\frac{(2mr - r_Q^2)a \sin^2\theta}{\Gamma}, \end{aligned}$$

with

$$\begin{aligned}\Delta &= r^2 - 2mr + a^2 + r_Q^2, \\ \Gamma &= r^2 + a^2 \cos^2 \theta,\end{aligned}$$

and $a = \frac{J}{m}$ and $r_Q = \frac{Q^2}{4\pi\epsilon_0}$. The horizon of Kerr-Newman metric is at $\Delta = 0$

$$r^2 - 2mr + a^2 + r_Q^2 = 0 \rightarrow r_{\pm} = m \pm \sqrt{m^2 - a^2 - r_Q^2}.$$

In order to derive the near horizon geometry of the Kerr-Newman metric we define a new variable

$$\lambda = \int_{r_+}^r \frac{dr'}{\Delta(r')} \approx 2\sqrt{\frac{r - r_+}{r_+ - r_-}} + O((r - r_+)^{3/2}).$$

Small values of λ correspond to radii close to the horizon r_+ . Replacing r in the metric with the new coordinate λ and keeping the leading terms for $\lambda \ll 1$, we obtain

$$\begin{aligned}\Delta &= (r - r_+)(r - r_-) \approx \frac{(r_+ - r_-)^2}{4} \lambda^2, \\ \Gamma &= r^2 + a^2 \cos^2 \theta \approx r_+^2 + a^2 \cos^2 \theta \equiv \Gamma_+, \\ g_{tt} - \frac{g_{t\phi}^2}{g_{\phi\phi}} &\approx -\frac{(r_+ - r_-)^2}{4} \frac{\Gamma_+ \lambda^2}{(r_+ + a^2)^2}, \\ g_{rr} &\approx \frac{4\Gamma_+}{(r_+ - r_-)^2 \lambda^2}, \\ g_{\theta\theta} &\approx \Gamma_+, \\ \frac{g_{t\phi}}{g_{\phi\phi}} &\approx -\frac{a}{r_+^2 + a^2} \equiv -\Omega, \\ g_{\phi\phi} &\approx \frac{(r_+^2 + a^2)^2 \sin^2 \theta}{\Gamma_+}.\end{aligned}$$

Defining new variables $\psi \equiv (\phi - \Omega t)(r_+^2 + a^2)$ and $\tau \equiv \frac{r_+ - r_-}{2(r_+^2 + a^2)} t$, we get

$$ds^2 = -\Gamma_+ \lambda^2 d\tau^2 + \Gamma_+ d\lambda^2 + \Gamma_+ d\theta^2 + \frac{\sin^2 \theta}{\Gamma_+} d\psi^2. \quad (\text{B1})$$
